

# Cosmological bounds on tachyonic neutrinos

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Recent time-of-flight measurements on muon neutrinos in the OPERA neutrino oscillation experiment have found anomalously short times compared to the light travel-times, corresponding to a superluminal velocity,  $v - 1 = 2.37 \pm 0.32 \times 10^{-5}$  in units where  $c = 1$ . We show that cosmological bounds rule out an explanation involving a Lorentz invariant tachyonic neutrino. At the OPERA energy scale, nucleosynthesis constraints imply  $v - 1 < 0.86 \times 10^{-12}$  and the Cosmic Microwave Background observations imply  $v - 1 < 7.1 \times 10^{-23}$ . The CMB limit on the velocity of a tachyon with an energy of 10 MeV is stronger than the SN1987A limit. Superluminal neutrinos that could be observed at particle accelerator energy scales would have to be associated with Lorentz symmetry violation.

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## I. INTRODUCTION

Recent time-of-flight measurements on muon neutrinos in the OPERA neutrino oscillation experiment have found anomalously short flight-times compared to the light-travel times [1]. The anomaly corresponds to a superluminal velocity,  $v - 1 = 2.37 \pm 0.32 \times 10^{-5}$  in units where  $c = 1$ . The simplest type of particle with superluminal velocities would be a tachyon, a Lorentz invariant particle with imaginary mass, although this possibility conflicts with the observations of (anti)neutrinos associated with the supernova SN 1987A, which imply  $v - 1 < 2 \times 10^{-9}$  for 10 MeV neutrinos [2–6]. We show that cosmological bounds support the supernova result and also rule out a tachyonic explanation of the neutrino timing anomaly. We find, at the OPERA energy scale,  $v - 1 < 0.86 \times 10^{-12}$  from nucleosynthesis constraints and  $v - 1 < 7.1 \times 10^{-23}$  from the Cosmic Microwave Background (CMB) observations. The CMB limit on the velocity of a tachyon with an energy of 10 MeV is stronger than the SN1987A limit.

## II. TACHYONS IN AN EXPANDING UNIVERSE

Our treatment of tachyonic particles in an expanding universe follows Refs. [7] and [8]. The tachyon is a particle with imaginary mass  $i\mu$  and speed  $v > 1$ , whose energy and momentum in a local inertial frame are given by

$$E = \mu(v^2 - 1)^{-1/2}, \quad (1)$$

$$\mathbf{p} = \mu\mathbf{v}(v^2 - 1)^{-1/2}. \quad (2)$$

We also find it useful to express the speed as a function of the energy,

$$v(E) = \left(1 + \frac{\mu^2}{E^2}\right)^{1/2}. \quad (3)$$

The tachyon speeds up if its energy is reduced.

Consider such a particle in a spatially-flat Friedmann-Robertson-Walker universe with cosmological time  $t$  and scale factor  $a(t)$ . If we use  $v^i$  to denote the velocity in the co-moving frame, then the conserved linear momentum is

$$p_i = \mu a^{-2} v^i. \quad (4)$$

The energy and momentum of the tachyon are related by

$$p_i p^i - E^2 = \mu^2. \quad (5)$$

The energy of a tachyon in an expanding universe is therefore given by

$$E = \mu \left( \frac{p_i p_i}{\mu^2} \frac{1}{a^2} - 1 \right)^{1/2}. \quad (6)$$

The same result was obtained in Ref. [8] using the law of addition of velocities in an expanding universe. The important feature here is that the energy of a non-interacting tachyon falls *faster* than the usual  $1/a$  rate for massless particles. At some point the energy falls to zero and the tachyons disappear from the universe. This may be interpreted as a tachyon-antitachyon annihilation event [7].

The foregoing analysis only applies to free tachyons. A tachyon in thermal equilibrium with ordinary particles at temperature  $T$  would have energy  $E \approx T$ , as usual. If the tachyons come out of equilibrium, then the constants in Eq. 6 are fixed by the energy at the relevant decoupling time.

## III. COSMOLOGICAL BOUNDS

We shall start off the cosmological bounds with nucleosynthesis constraints (see e.g. [9] and [10] for a review), and focus on events between neutrino decoupling at a temperature  $T_{\text{weak}}$  and element formation at  $T_{\text{nuc}}$ . An effective neutrino number  $N_\nu$  is defined in terms of the energy densities of neutrinos  $\rho_\nu$  and photons  $\rho_\gamma$ . For times preceding electron-positron annihilation,

$$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8} N_\nu. \quad (7)$$

There is an extra factor of  $(4/11)^{4/3}$  on the right-hand-side after electron-positron annihilation. In the standard model, with three lepton families, we have  $N_\nu = 3$ .

The cosmological expansion rate is sensitive to the number of neutrino species, and therefore both the neutrino decoupling time and the neutron/proton ratio at the time of helium formation are dependent on  $N_\nu$ . By modelling the primordial element abundances and comparing them to observations it is possible to set limits on  $\Delta N_\nu = N_\nu - 3$ . Typical examples of these limits are  $\Delta N_\mu > -1.7$  (95% CL) [10] and  $\Delta N_\mu > -1.0$  (95% CL) [9]. Note that these results assume  $\Delta N_\mu$  is constant from decoupling to helium formation.

Free tachyonic neutrinos, the energy is given by Eq. (6) with  $T \propto 1/a$ , and we have  $E_\nu = T_{\text{weak}}$  at neutrino decoupling to fix the constants,

$$E_\nu(T) = \mu \left( \frac{T^2}{T_{\text{weak}}^2} + \frac{T^2}{\mu^2} - 1 \right)^{1/2}. \quad (8)$$

Neutrino oscillation experiments imply that all three neutrino species would have a similar tachyonic mass [11]. The neutrino energy density  $\rho_\nu \propto T^3 E_\nu$  and the photon energy density  $\rho_\gamma \propto T^4$ , so that the effective number of neutrino species defined by Eq. (7) is

$$N_\nu(T) = 3 \left( 1 + \frac{\mu^2}{T_{\text{weak}}^2} - \frac{\mu^2}{T^2} \right)^{1/2}. \quad (9)$$

Note that  $N_\nu = 3$  at neutrino decoupling, then it decreases to zero and remains zero after the tachyon annihilation.

Given limits on  $\Delta N_\nu$  at the nucleosynthesis time, then we can invert (9) to set limits on the tachyon mass

$$\mu^2 \leq \frac{2}{3} \frac{T_{\text{nuc}}^2 T_{\text{weak}}^2}{T_{\text{weak}}^2 - T_{\text{nuc}}^2} |\Delta N_\nu|. \quad (10)$$

Typical values would be  $T_{\text{weak}} = 0.8\text{MeV}$  for decoupling,  $T_{\text{nuc}} = 0.1\text{MeV}$  for light element formation and  $|\Delta N_\nu| < 2$ ,

$$\mu \leq 0.12\text{MeV}. \quad (11)$$

The bounds on the tachyonic mass translate into bounds on the speed of the tachyon at a given energy scale via Eq. (3). If we were to compare with the OPERA results [1], at an energy of  $28\text{GeV}$ ,

$$v(28\text{GeV}) - 1 < 0.86 \times 10^{-12}. \quad (12)$$

The speed reported by the OPERA announcement corresponds to  $v - 1 = 2.4 \times 10^{-5}$ .

CMB observations can be used to give information about the neutrino density of the universe at later times than the nucleosynthesis era. The acoustic oscillations in the CMB spectrum are sensitive to the free-streaming of the neutrinos prior to recombination time. The WMAP 7-year data analysis, assuming a  $\Lambda\text{CDM}$  model, places a limit  $N_\nu > 2.7$  (95% CL) [12, 13]. An earlier analysis of the WMAP 5-year data using independent input on the values of the expansion rate  $H_0$  and the parameter  $\sigma_8$  gave a similar limit  $N_\nu > 2.55$  (95% CL) [14].

Consider the latest limit with  $\Delta N_\nu > -0.3$ . We can replace  $T_{\text{nuc}}$  in Eq. (10) by the temperature of the universe at matter-radiation equality when the density perturbations start to grow,  $T_{\text{eq}} = 0.74\text{eV}$ . The limit on the tachyon mass becomes

$$\mu \leq 0.33\text{eV}. \quad (13)$$

The limits on the speed of the neutrinos at  $28\text{GeV}$  become

$$v(28\text{GeV}) - 1 < 7.1 \times 10^{-23}. \quad (14)$$

We also find that  $v(10\text{MeV}) - 1 < 5.4 \times 10^{-16}$ , which is better than the supernova SN1987A limits on the neutrino velocity, although this only applies to the case of Lorentz invariant tachyons. With such small tachyonic masses, the three neutrino species could have a combination of tachyonic and real masses and still be consistent with neutrino oscillation experiments (which measure differences in  $\mu^2$ ), but this does not affect the bound.

#### IV. CONCLUSION

We conclude that the cosmological bounds rule out the possibility of a tachyonic neutrino over a wide range of tachyonic masses and the limits are better than those obtained from supernova 1987A. As a consequence, a  $\text{GeV}$

energy-scale neutrino that travels appreciably faster than the speed of light would need to be associated with a violation of Lorentz symmetry. It would be possible to take specific models of Lorentz symmetry violation (see [15, 16] for reviews) and repeat the cosmological analysis to obtain useful limits on these theories.

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- [1] T. Adam *et al.* (OPERA Collaboration), (2011), arXiv:1109.4897 [hep-ex].
  - [2] K. Hirata *et al.* (KAMIOKANDE-II Collaboration), Phys.Rev.Lett. **58**, 1490 (1987).
  - [3] R. Bionta, G. Blewitt, C. Bratton, D. Casper, A. Ciocio, *et al.*, Phys.Rev.Lett. **58**, 1494 (1987).
  - [4] L. Stodolsky, Phys.Lett. **B201**, 353 (1988).
  - [5] M. J. Longo, Phys.Rev.Lett. **60**, 173 (1988).
  - [6] G. Cacciapaglia, A. Deandrea, and L. Panizzi, JHEP **1111**, 137 (2011), arXiv:1109.4980 [hep-ph].
  - [7] P. C. Davies, Nuovo Cimento **25B** (1975).
  - [8] P. C. Davies, Int.J.Theor.Phys. **43**, 141 (2004), arXiv:astro-ph/0403048 [astro-ph].
  - [9] B. Fields and S. Sarkar, J.Phys.G **G33**, 1 (2006), in the Review of Particle Properties, arXiv:astro-ph/0601514 [astro-ph].
  - [10] G. Steigman, Int.J.Mod.Phys. **E15**, 1 (2006), arXiv:astro-ph/0511534 [astro-ph].
  - [11] M. Gonzalez-Garcia and M. Maltoni, Phys.Rept. **460**, 1 (2008), arXiv:0704.1800 [hep-ph].
  - [12] D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. Nolte, *et al.*, Astrophys.J.Suppl. **192**, 16 (2011), arXiv:1001.4635 [astro-ph.CO].
  - [13] E. Komatsu *et al.* (WMAP Collaboration), Astrophys.J.Suppl. **192**, 18 (2011), arXiv:1001.4538 [astro-ph.CO].
  - [14] B. A. Reid, L. Verde, R. Jimenez, and O. Mena, JCAP **1001**, 003 (2010), arXiv:0910.0008 [astro-ph.CO].
  - [15] D. Mattingly, Living Reviews in Relativity **8** (2005).
  - [16] S. Liberati and L. Maccione, Ann.Rev.Nucl.Part.Sci. **59**, 245 (2009), arXiv:0906.0681 [astro-ph.HE].